Name of college – S. S. College J-Bad

Deptt – Mathematic

Topic – Length of Normal Subnormal, Tangent, Sub-tangent.

Class – B.Sc I (Hons)

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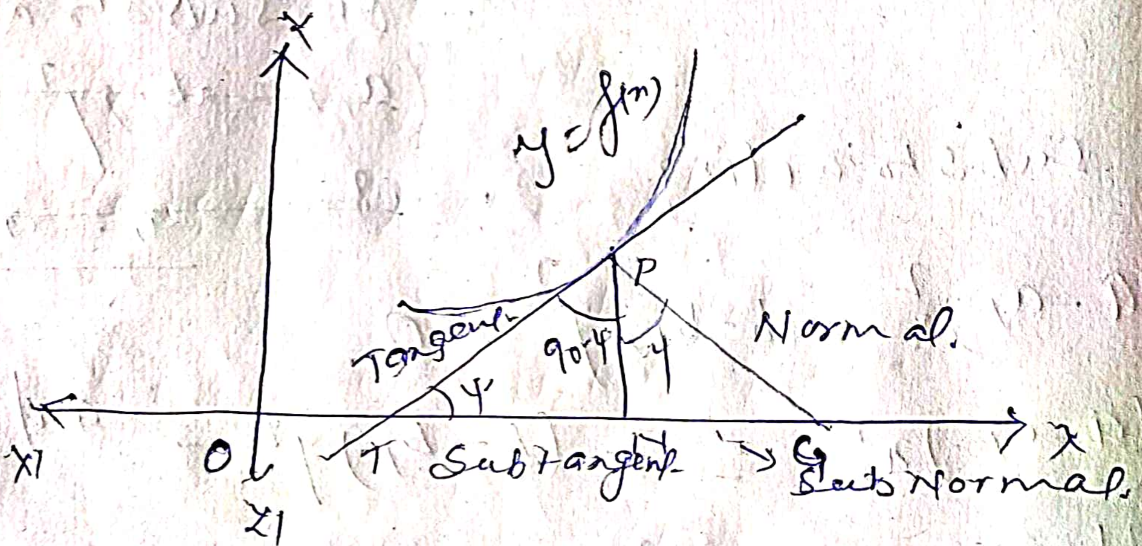
Study Material



Let $y = f(x)$ be the equation of curve. Let $P(x, y)$ be any point on the curve.

Here PN is drawn perpendicular to x-axis $\Rightarrow ON = x$ and $PN = y$

Draw PT tangent to the curve which meet x-axis at T and PG Normal to the curve which meet x-axis at G

Projection of PT on x-axis = TN = Sub Tangent.
Projection of PG on x-axis = NG = Sub Normal

$\angle TPG = \pi/2$ $\angle PTX = \psi$

PT = Length of Tangent.
PG = Length of ~~sub~~ normal.

Now clearly $\angle NPG = \psi$

From $\triangle PNT$

$\cancel{\frac{y}{PT} = \sin\psi}$ $\tan\psi = \frac{y}{TN}$

$\Rightarrow TN = \frac{y}{\tan\psi} = \frac{y}{\frac{dy}{dx}} = \frac{y}{y_1}$

Length of Sub tangent $= y/y_1$

Again from $\triangle PNG$

$$\frac{NG}{PN} = \tan\psi$$

$\Rightarrow NG = PN \tan\psi$
$= y \cdot y_1$ $\left(\text{Where } y_1 = \frac{dy}{dx}\right)$

$\Rightarrow$ Length of Sub Normal $= y \cdot y_1$

Also $PT^2 = PN^2 + NT^2$

$= y^2 + \frac{y^2}{y_1^2}$

$= y^2\left(1 + \frac{1}{y_1^2}\right) = \frac{y^2}{y_1^2}\left(1 + y_1^2\right)$

$\therefore PT$ = length of Tangent $= \frac{y}{y_1}\sqrt{1+y_1^2}$

$PG^2 = PN^2 + NG^2$

$= y^2 + y^2 \cdot y_1^2$

$= y^2(1 + y_1^2)$

$\therefore PG =$ Length of Normal

$= y\sqrt{1+y_1^2}$

Summary → Length of Tangent

$= \frac{y}{y_1}\sqrt{1+y_1^2}$

Length of Normal

$= y\sqrt{1+y_1^2}$

Length of subtangent

$= \frac{y}{y_1}$

Length of subnormal

$= y y_1$

Here Tangent and Normal are perpendicular

Projection of Tangent on x-axis = sa

Projection of Normal on x-axis = sa

if $y = f(x) \Rightarrow y_1 = \frac{dy}{dx} = f'(x)$

Problem based on Length of Tangent.
Length of Normal
Length of Sub tangent.
Length of Sub normal

Problem 1 → Show that in the Curve $y = be^{-\frac{a}{x}}$, the subtangent Varies as the square of abscissa.

Solution
Length of subtangent $= \frac{y}{y_1}$ Where $y_1 = \frac{dy}{dx}$ Where $y = f(x)$ is the Equation of curve.

Here $y = be^{-\frac{a}{x}}$

$$y_1 = be^{-a/x}\,(-a)\left(-\frac{1}{x^2}\right)$$

$$= be^{-a/x}\,\frac{a}{x^2}$$

$\therefore$ Length of Subtangent $= \frac{y}{y_1}$

$$= \frac{\cancel{be^{-(a/x)}}}{\cancel{be^{-a/x}}\,\frac{a}{x^2}}$$

$$= \frac{1}{a}x^2$$

$\Rightarrow$ Subtangent $\propto x^2$

Problem 2 :

Show that in the curve

$$by^2 = (a+x)^3$$

the square of subtangent ∝ subnormal

Solution → Here we are to show that

$$\frac{(\text{Square of subtange}}{\text{Subnormal}} = \text{Constant}$$

Equation of curve

$$by^2 = (a+x)^3$$

we have, subtangent $= \frac{y}{y_1}$

Subnormal $= y y_1$.

Now Equation of curve

$$by^2 = (a+x)^3$$

$$y^2 = \frac{(a+x)^3}{b}$$

Diff with respect to x

$$2y\frac{dy}{dx} = \frac{3(a+x)^2}{b}$$

$$\therefore \frac{dy}{dx} = \frac{3}{2y}\frac{(a+x)^2}{b} = y_1$$

Subtangent $= \frac{y}{y_1} = \frac{y}{\frac{3}{2by}(a+x)^2}$

$$= \frac{2by^2}{3(a+x)^2}$$

$$= \frac{2b\cdot y^2}{3(a+x)^2}$$

Subnormal $= y\,y_1$

$$= y\cdot\frac{3}{2y\cdot b}(a+x)^2$$

$$\frac{(\text{Subtangent})^2}{\text{Subnormal}} = \left[\frac{2by^2}{3(a+x)^2}\right]^2 \times \frac{2b}{3(a+x)^2}$$

$$= \frac{8b^2y^4}{27(a+x)^6}$$

$$= \frac{8}{27}\frac{(by^2)^2}{(a+x)^6} = \frac{8b}{27}\frac{(a+x)^6}{(a+x)^6}$$

$$= \frac{8}{27}b = \text{constant}$$

Problem 3. Show that the Sub-tangent at any point of the curve $x^m y^n = a^{m+n}$ varies as the abscissa.

Solution :→ Sub tangent $= \frac{y}{y_1}$ — ①

Where $y = f(x)$. is the equation of curve

and $y_1 = \frac{dy}{dx}$

Here Equation of the curve is

$$x^m y^n = a^{m+n}$$

Diff. with respect to x

$$m x^{m-1} y^n + n x^m y^{n-1} \frac{dy}{dx} = 0$$

$$\Rightarrow n x^m y^{n-1} \frac{dy}{dx} = -m x^{m-1} y^n$$

$$\therefore \frac{dy}{dx} = -\frac{m x^{m-1} y^n}{n x^m \cdot y^{n-1}}$$

$$= -\frac{m}{n} \frac{y}{x}$$

Now Subtangent $= \frac{y}{y_1}$

$$= \cancel{y} \times \frac{1}{-\frac{m}{n}\frac{\cancel{y}}{x}}$$

$$= -\frac{n}{m}\cdot x$$

∴ Subtangent

⇒ Subtangent $\propto x$

thus Subtangent at any point (x, y) is varies as abscissa.